Int. J. Heat Mass Transfer. Vol. 29, No. 1, pp. 107–117, 1986
Printed in Great Britain

0017-9310/86 \$3.00 + 0.0

Convective heat transfer for laminar flows in a multi-passage circular pipe subjected to an external uniform heat flux

M. A. EBADIAN,* H. C. TOPAKOGLU* and O. A. ARNAS†

*Department of Mechanical Engineering, Southern University, Baton Rouge, LA 70813, U.S.A.
†Department of Mechanical Engineering, Louisiana State University, Baton Rouge, LA 70803, U.S.A.

(*Received* 21 *January* 1985 *and in final form* 25 *July* 1985)

Abstract—Convective heat transfer properties of an hydrodynamically and thermally fully-developed flow in a multi-passage circular tube subjected to an external uniform heat flux are analyzed. The significant dimensionless parameters affecting the problem have been determined. The expressions of Nusselt numbers on the inner and the outer wall are obtained. The limiting cases are correlated with the existing values. The representative curves illustrating the variation of Nusselt numbers with the pertinent parameters are plotted.

1. INTRODUCTION

THE EXISTING heat transfer characteristics of laminar flow in a circular pipe subjected to an external uniform heat flux is extended, without simplifying assumptions, to the case of combinations of laminar multi-passage flows maintained under the same boundary conditions. Neglecting the heat conduction effect at the inner separation wall, the temperature distribution field is obtained at any section of the segment of the system where flows are hydrodynamically and thermally fully developed. All possible approaches for obtaining the dimensionless heat transfer coefficients, the Nusselt numbers, at the interface which separates the inner and outer flow, and at the outer wall are systematically analyzed. It is shown that the Nusselt numbers depend only on two dimensionless parameters:

(i) The ratio of the thermal conductivities of the fluids from the inner and outer passages, k_k.
(ii) The product of k_k and the ratio of the Péclet numbers of the flows from the inner and outer passages, $\eta = k_k k_{Pe}$, which is here called the heat exchange number of the multi-passage pipe.

Corresponding to each of the approaches, the analytical forms of Nusselt numbers are obtained. Typical values for the ratio of the thermal conductivities of the two fluids and the heat exchange numbers are selected. The Nusselt numbers are then plotted against the dimensionless radius of the interface. The gradual effects of the velocity changes in the inner and the outer passages are demonstrated by plotting the Nusselt numbers against the heat exchange number for a typical ratio of heat conductivities and a typical value of dimensionless radius of interface separation. The physical significances of the Nusselt numbers corresponding to the limiting cases of the interface located at the outer wall and of the interface shrinking to a line at the center are correlated to the known values.

2. TEMPERATURE DISTRIBUTION

A literature survey [1–7] indicates that convective heat transfer studies for laminar flows in multi-passage circular pipes based on exact temperature distributions are not available. The interest for wall heat transfer coefficients based on exact temperature distribution is of importance, for both the theory and the practical design considerations of heat exchangers involving two flows in separated sections of a conduit. In this paper, the convective heat transfer characteristics of two such flows, one confined in the inner passage and one confined in the other passage of a circular pipe subjected to an arbitrary longitudinal external heat flux, are analyzed. The thickness of the separating interface and the heat conduction resistance of the inner wall, however, are neglected. The flow characteristics and the notations used are shown in Fig. 1.

The differential equations for the velocities of the outer and the inner flows which are maintained under independent pressure gradients are, respectively,

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right) W_o = \frac{1}{\mu_o}\frac{\partial P_o}{\partial Z},$$
$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right) W_i = \frac{1}{\mu_i}\frac{\partial P_i}{\partial Z} \quad (1)$$

where X and Y are transversal rectangular coordinates and P_o, P_i are the pressures in the outer and inner passages, respectively.

The Reynolds numbers for each flow are defined as the Reynolds numbers for a simple circular pipe of radius L filled with the corresponding fluid and maintained under the corresponding pressure gradient in each passage [8]. Thus,

$$(Re)_o = -\frac{1}{4}\frac{L^3}{\mu_o \nu_o}\frac{\partial P_o}{\partial Z}, \quad (Re)_i = -\frac{1}{4}\frac{L^3}{\mu_i \nu_i}\frac{\partial P_i}{\partial Z}. \quad (2)$$

It must be noted that the ratio between the Reynolds numbers represents the ratio between the pressure

NOMENCLATURE

A_o, A_i cross-sectional areas for outer and inner passages, respectively [m^2]
C, c dimensional and dimensionless temperature gradient along the pipe, equation (12)
C_{po}, C_{pi} specific heats of outer and inner flows, respectively
E_i, e_i dimensional and dimensionless excess temperature of inner pipe, equation (12)
E_{in}, e_{in} dimensional and dimensionless interface temperature, equation (12)
E_o, e_o dimensional and dimensionless excess temperature of outer pipe, equation (12)
E_{mo}, E_{mi} mixed mean excess temperature of the outer and inner pipe, respectively, equation (25) [K]
F characteristic heat flux, equation (12)
f_o factor defined by equation (14)
G_o, G_i dimensionless function for outer and inner fluids, respectively, equations (14) and (15)
g factor defined by equation (14)
H_o, h_o dimensional and dimensionless heat generation densities of outer fluid, equation (12)
H_i, h_i dimensional and dimensionless heat generation densities of inner fluid, equation (12)
$\bar{h}_o, \bar{h}_i$ outer and inner convective heat transfer coefficients, equations (34) [$W m^{-2} K^{-1}$]
I_{oo} function defined by equation (7)
I_{o1} function defined by equation (30)
J_o, J_i functions defined by equation (28)
J_{oo} function defined by equation (31)
k_o, k_i thermal conductivities of outer and inner fluids, respectively [$W m^{-1} K^{-1}$]
k_c ratio of the specific heats of the inner fluid to the outer fluid, equation (26)
k_k ratio of the thermal conductivities of the inner fluid to the outer fluid, equation (38)
k_{Pe} ratio of the Péclet numbers of the inner fluid to those of the outer fluid
L outer radius, equation (3) and Fig. 1 [m]
$(Nu)_o, (Nu)_i$ Nusselt numbers on the outer and inner walls, respectively, equations (35)
P_o, P_i pressure for outer and inner passages, equation (1) [kPa]
$(Pe)_o, (Pe)_i$ Péclet numbers, $Re\,Pr$, of outer and inner fluids, respectively
$(Pr)_o, (Pr)_i$ Prandtl numbers of outer and inner fluids, respectively, ν/α
Q_o, Q_i mass flow rates of outer and inner fluids, respectively, equation (6) [$kg\, s^{-1}$]
R, r dimensional and dimensionless radial coordinates for both flow regions varying between $R = r = \omega$, the inner pipe and $R = r = 1$, the outer pipe, Fig. 1
$(Re)_o, (Re)_i$ Reynolds number of outer and inner fluids, respectively, equation (2)
T_o, T_i outer and inner wall temperatures [K]
T_m mixed mean temperature, equation (23) [K]
U_o, U_i heat fluxes from outer and inner walls, equation (20)
$\bar{W}$ velocity vector [$m\, s^{-1}$]
W_i, w_i dimensional and dimensionless velocity of inner fluid, equation (3)
W_o, w_o dimensional and dimensionless velocity of outer fluid, equation (3)
X transversal rectangular coordinate
Y transversal rectangular coordinate; also function defined by equation (29)
Z dimensional longitudinal coordinate.

Greek symbols
α thermal diffusivity, [$m^2\, s^{-1}$]
β function defined by equation (17)
λ ratio between the mass flow rates, equation (8)
η dimensionless heat exchange number, equation (19)
μ function defined by equation (5)
μ_o, μ_i coefficients of dynamic viscosity of outer and inner fluids, respectively, [$kg\, m^{-1}\, s^{-1}$]
ν_o, ν_i coefficients of kinematic viscosities of outer and inner fluids, respectively [$m^2\, s^{-1}$]
ρ_o, ρ_i densities of inner and outer fluids, respectively [$kg\, m^{-3}$]
ω dimensionless inner radius of the annulus, Fig. 1

gradients of the flows in the separated regions and their respective viscosities.

Using the kinematic viscosity ν_o as a reference viscosity and defining the dimensionless radial coordinate and the dimensionless velocities by the relations,

$$R = Lr, \quad W_o = \frac{\nu_o}{L} w_o, \quad W_i = \frac{\nu_o}{L} w_i \tag{3}$$

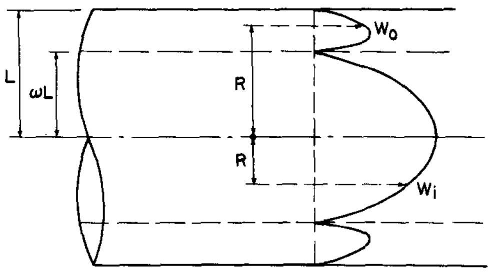

FIG. 1. The annular geometry.

the dimensionless velocities for the flows in the outer and the inner passages are:

$$w_o = (Re)_o(1-r^2+\mu\ln r),\quad w_i = \frac{v_i}{v_o}(Re)_i(\omega^2-r^2) \tag{4}$$

where ω is the dimensionless interface radius and

$$\mu = -(1-\omega^2)/\ln\omega. \tag{5}$$

The mass rates of flow for each section, which will later enter into the formulation of the heat transfer coefficients, are

$$Q_o = 2\pi\mu_o L(Re)_o I_{oo},\quad Q_i = \frac{\pi}{2}\mu_i\omega^4 L(Re)_i \tag{6}$$

where

$$I_{oo} = \frac{1}{4}(1-\omega^2)(1+\omega^2-\mu). \tag{7}$$

The ratio between the mass flow rates is

$$\lambda = \frac{Q_i}{Q_o} = \frac{1}{4}\frac{\mu_i}{\mu_o}\frac{(Re)_i}{(Re)_o}\frac{\omega^4}{I_{oo}}. \tag{8}$$

At sufficient distances from both entrances, the fully-developed temperature distributions are established and the temperature distributions in the outer and inner passages due to convective heat transfer caused by the external uniform heat flux condition have the following functional forms [8]:

$$T_o = CZ+E_o(X,Y),\quad T_i = CZ+E_{in}+E_i(X,Y) \tag{9}$$

where C is the uniform external temperature gradient along the pipe, E_o and E_i are the excess temperatures of the outer and inner flows, respectively, and E_{in} is the interface temperature. The boundary conditions of excess temperatures are

$$\begin{aligned} &E_o = 0 && \text{on the outer periphery}\\ &E_o = E_{in} && \text{on the interface}\\ &E_i = 0 && \text{on the interface.}\end{aligned} \tag{10}$$

The energy equation to be satisfied by each temperature distribution, in order to maintain generality, including a uniform heat generation H, is

$$\bar{V}\cdot\text{grad}\,T = \alpha\left(\nabla^2 T+\frac{H}{k}\right) \tag{11}$$

where $\bar{V}$ is the velocity vector, T is temperature and α is thermal diffusivity for the proper flows in each passage.

The dimensionless thermal variables are introduced by the following relations,

$$\begin{aligned} &H_o = \frac{F}{L}h_o,\quad C = \frac{F}{k_o}c,\quad E_o = \frac{LF}{k_o}e_o\\ &H_i = \frac{F}{L}h_i,\quad E_{in} = \frac{LF}{k_o}e_{in},\quad E_i = \frac{LF}{k_o}e_i\end{aligned} \tag{12}$$

where H_o, H_i and h_o, h_i are dimensional and dimensionless heat generation densities for outer and inner passage flows, respectively. F denotes an arbitrary heat flux.

After some similar calculations, as was done in [8], the dimensionless temperatures for the outer passage and the inner passage are obtained as follows.

The outer flow dimensionless excess temperature:

$$e_o = e_{in}g+h_o f_o+G_o e_{oo} \tag{13}$$

where

$$\begin{aligned} &G_o = c(Pe)_o,\quad (Pe)_o = \text{Péclet number of outer flow}\\ &g = (\ln r)/(\ln\omega),\quad f_o = \frac{1}{4}(1-r^2+\mu\ln r)\\ &e_{oo} = e_{01}+e_{02}+e_{03}+e_{04}+e_{05}\\ &e_{01} = -\frac{1}{4}(1-r^2+\mu\ln r)\\ &e_{02} = 0\\ &e_{03} = \frac{1}{4}\mu\left(1-\frac{\omega^2}{r^2}\right)r^2\ln r\\ &e_{04} = \frac{1}{16}[1-r^2+\mu(1+\omega^2)\ln r]\\ &e_{05} = \frac{1}{4}\mu(1-r^2+\mu\ln r).\end{aligned} \tag{14}$$

The zero value of the term e_{02} is maintained here for easy cross reference to [8]. It will not vanish for a section of elliptic form.

The inner flow dimensionless excess temperature is given as:

$$e_i = \frac{1}{16}(\omega^2-r^2)\left[4\frac{k_o}{k_i}h_i-G_i\frac{1}{\omega^3}(3\omega^2-r^2)\right] \tag{15}$$

where $G_i = c\omega^3(Pe)_i$, $(Pe)_i$ = Péclet number of inner flow.

The value of the dimensionless interface temperature, e_{in}, will be determined by the condition of heat flux continuity at the interface as

$$k_o\left(\frac{de_o}{dr}\right)_{r=\omega} = k_i\left(\frac{de_i}{dr}\right)_{r=\omega}. \tag{16}$$

Introducing an alternate dimensionless interface temperature β as $\beta = e_{in}/G_o$ and substituting e_o and e_i

from (13) and (15) into (16), after neglecting the heat generations, one finds

$$\beta = \left[\frac{1}{16}\frac{\mu}{\omega}(3-4\mu+7\omega^2)-\frac{1}{4}\omega^3+\frac{1}{4}\frac{k_i}{k_o}\frac{G_i}{G_o}\right]\omega\ln\omega. \tag{17}$$

Substitution of G_o and G_i into (17) yields

$$\beta = \left[\frac{1}{16}\mu(3-4\mu+7\omega^2)-\frac{1}{4}(1-\eta)\omega^4\right]\ln\omega. \tag{18}$$

where

$$\eta = \frac{k_i}{k_o}\frac{(Pe)_i}{(Pe)_o} = k_k k_{Pe}. \tag{19}$$

Due to the fact that the heat flow crossing the interface between the outer and inner flows will be affected by η, this dimensionless factor can be called the heat exchange number of the multi-passage flow. Furthermore the sign of this parameter determines the relative directions of the flows. That is, when η is positive, the flows are unidirectional, parallel-flow combination; when η is negative, the flows are in opposite directions, the counter-flow combination.

3. HEAT FLUXES AND HEAT TRANSFER COEFFICIENTS

The rates of heat flows per unit length of pipe through the outer and inner surfaces, considered positive when flowing into the outer section are expressed as [8]:

$$U_o = 2\pi LFG_o\left[\frac{\beta}{\ln\omega}+e'_{oo}(1)\right]$$

$$U_i = -2\pi LFG_o\left[\frac{\beta}{\ln\omega}+\omega e'_{oo}(\omega)\right] \tag{20}$$

where

$$e'_{oo}(1) = \frac{1}{4}(1-\mu)-\frac{3}{16}\mu(1+\omega^2)+\frac{1}{4}\mu^2$$

$$\omega e'_{oo}(\omega) = e'_{oo}(1)-I_{oo}. \tag{21}$$

Here, the primes indicate differentiation with respect to r.

By substituting the excess temperature, e_o, from (13) and the excess temperature, e_i, from (15), the rates of heat flows are simplified as

$$U_o = 2\pi LFG_o\frac{1}{4}[(1-\omega^2)(1+\omega^2-\mu)+\eta\omega^4]$$

$$U_i = -2\pi LFG_o\frac{1}{4}\eta\omega^4. \tag{22}$$

The existence of two separate independent flow regions makes it possible for the surface heat transfer coefficients to be defined in various ways. The three approaches considered in this paper are:

Case I. The inner and outer wall transfer coefficients are defined relative to the mixed flow bulk temperature of the inner and outer flow.

Case II. The heat transfer coefficient at the inner surface of the interface is defined relative to the bulk temperature of the inner flow. The heat transfer coefficient at the outer surface of the interface is defined relative to the bulk temperature of the outer flow. In this case, since the inner flow is also subjected to an external uniform heat flux, the former heat transfer coefficient will have a trivial value corresponding to the usual dimensionless number 48/11.

Case III. The heat transfer coefficient at the inner surface is defined relative to the difference between the outer flow and inner flow bulk temperatures.

The mixed mean temperature of the combined flow is

$$T_m = \left(C_{po}\rho_o\int_{A_o} W_oT_o\,dA + C_{pi}\rho_i\int_{A_i} W_iT_i\,dA\right)\Big/(C_{po}Q_o+C_{pi}Q_i) \tag{23}$$

where ρ_o, A_o and ρ_i, A_i are the densities and the cross-sectional areas for the outer and inner passages, respectively.

Substituting T_o and T_i from (9), this reduces to

$$T_m = CZ+(E_{mo}+k_c\lambda E_{in}+k_c\lambda E_{mi})\left(\frac{1}{1+k_c\lambda}\right) \tag{24}$$

where

$$E_{mo} = \frac{\rho_o}{Q_o}\int_{A_o} W_oE_o\,dA,\quad E_{mi} = \frac{\rho_i}{Q_i}\int_{A_i} W_iE_i\,dA \tag{25}$$

and

$$k_c = C_{pi}/C_{po}. \tag{26}$$

Substituting the dimensional variables for the dimensionless counterparts, the mixed mean excess temperatures E_{mo} and E_{mi} are expressed as

$$E_{mo} = -\frac{FL}{k_o}\frac{J_o}{I_{oo}},\quad E_{mi} = -\frac{FL}{k_o}\frac{4}{\omega^4}J_i \tag{27}$$

where

$$J_o = -\frac{1}{(Re)_o}\int_\omega^1 rw_oe_o\,dr,$$

$$J_i = -\frac{1}{(Re)_i}\frac{\nu_o}{\nu_i}\int_0^\omega rw_ie_i\,dr. \tag{28}$$

After performing the integrations for J_o and J_i, one finds

$$\frac{J_o}{G_o} = \beta Y+J_{oo},\quad Y = -\frac{1}{\ln\omega}I_{01} \tag{29}$$

where

$$I_{01} = -\frac{3}{16}(1-\omega^4)-\frac{1}{4}(1-\omega^2)\omega^2 + \frac{1}{4}(1-\omega^2)\mu-\frac{1}{4}\omega^4\ln\omega$$

$$J_{oo} = \frac{1}{48}\left[\frac{11}{8}(1-\omega^8) - \frac{19}{6}(1-\omega^6)\mu - \frac{27}{16} \times (1+\omega^2)(1-\omega^4)\mu + \frac{51}{8}(1-\omega^4)\mu^2 - 3(1-\omega^2)\mu^3\right]. \tag{30}$$

$$\frac{J_i}{G_o} = \frac{11}{384}\omega^8 k_{Pe}. \tag{31}$$

The individual bulk temperatures of the outer and inner flows are

$$T_{mo} = \frac{\rho_o}{Q_o}\int_{A_o} W_o T_o \, dA, \quad T_{mi} = \frac{\rho_i}{Q_i}\int_{A_i} W_i T_i \, dA \tag{32}$$

Substituting the expressions for T_o and T_i from (9) into (32) gives

$$T_{mo} = CZ + E_{mo}, \quad T_{mi} = CZ + E_{in} + E_{mi} \tag{33}$$

where E_{mo} and E_{mi} are the same expressions as in the previously written forms in (27).

Case I

If the heat transfer coefficients ($\bar{h}_o$ and $\bar{h}_i$) at the outer and inner surfaces are defined relative to the overall bulk temperature of the entire section, one can write

$$U_o = (CZ - T_m)2\pi L\bar{h}_o, \quad U_i = (CZ + E_{in} - T_m)2\pi\omega L\bar{h}_i. \tag{34}$$

Selecting the fluid in the outer passage as the reference fluid, the Nusselt numbers on each surface based on their diameters are

$$(Nu)_o = \frac{2L}{k_o}\bar{h}_o \quad \text{and} \quad (Nu)_i = \frac{2\omega L}{k_o}\bar{h}_i. \tag{35}$$

Equating the two forms of the rates of heat flows in (22) and (34), and neglecting heat generations, the result simplifies to give

$$(1-\omega^2)(1+\omega^2-\mu) + \eta\omega^4 = \frac{8}{4I_{oo}+\eta\omega^4} \times \left[\frac{J_o}{G_o} + \frac{1}{4}\eta\omega^4\left(\frac{11}{96}\frac{\eta}{k_k}\omega^4 - \beta\right)\right](Nu)_o \tag{36}$$

and

$$\eta\omega^4 = -\frac{8}{4I_{oo}+\eta\omega^4} \times \left[\frac{J_o}{G_o} + \frac{1}{4}\eta\omega^4\left(\frac{11}{96}\frac{\eta}{k_k}\omega^4\right) + \beta I_{oo}\right](Nu)_i. \tag{37}$$

where

$$k_k = k_i/k_o \tag{38}$$

and $I_{oo}, \beta, J_0/G_0$ are given by equations (7), (18) and (29), respectively.

It is to be noted that the Nusselt numbers in (36) and (37), besides being dependent on the size of the interface, depend only on the following two dimensionless parameters:

(i) the ratio of the thermal conductivities of two fluids, k_k;
(ii) the heat exchange number, η.

One advantage of formulating the Nusselt numbers as in Case I is that the Nusselt number at the outer surface in both limiting cases of $\omega = 0$ and $\omega = 1$ reduces to the expected value of 48/11. Also, the Nusselt number at the interface in the limiting case of $\omega = 1$ reduces to 48/11, and in the limiting case of $\omega = 0$ to zero.

Case II

If the heat transfer coefficients at each of the surfaces are defined relative to the mutual bulk temperatures of each of the separated flows, considering the positive heat flow into the outer region, one can write

$$U_o = (CZ - T_{mo})2\pi L\bar{h}_{oo}$$

$$U_i = (CZ + E_{in} - T_{mo})2\pi\omega L\bar{h}_{io}$$

(for the outer face of interface)

$$U_i = -(CZ + E_{in} - T_{mi})2\pi\omega L\bar{h}_{ii}$$

(for the inner face of interface). (39)

Selecting the fluid in the outer passage as the reference fluid, the Nusselt numbers on each of the surfaces based on their mutual diameter sizes are

$$(Nu)_o = \frac{2L}{k_o}\bar{h}_{oo},$$

$$(Nu)_{io} = \frac{2\omega L}{k_o}\bar{h}_{io},$$

$$(Nu)_{ii} = \frac{2\omega L}{k_o}\bar{h}_{ii}. \tag{40}$$

For this formulation, as was pointed out before, the Nusselt number $(Nu)_{ii}$ will produce a trivial value of 48/11.

Equating the two forms of the rates of heat flows in (22) and (39), and neglecting heat generations, the result simplifies to give

$$(1-\omega^2)(1+\omega^2-\mu) + \eta\omega^4 = 2\frac{J_o/G_o}{I_{oo}}(Nu)_{oo} \tag{41}$$

and

$$\eta\omega^4 = -2\left(\frac{J_o/G_o}{I_{oo}} + \beta\right)(Nu)_{io} \tag{42}$$

where I_{oo}, β, J_o/G_o are given in (7), (18) and (29), respectively.

In this case, it is seen that the two non-trivial Nusselt numbers, $(Nu)_{oo}$ and $(Nu)_{io}$, depend on only one single dimensionless parameter, the heat exchange number η. However, in both the limiting cases of $\omega = 0$ and $\omega = 1$, they will not reduce to the expected value of 48/11 and zero. Moreover, since both Nusselt numbers are based on the outer flow bulk temperature, their values will be identical to the Nusselt numbers for the inner and outer surfaces of a separate annular flow under proper surface heating conditions provided that Nusselt numbers are based on the hydraulic diameter. The ratio of these surface heating conditions can be obtained from (22) as

$$-\frac{U_i}{U_o}=\frac{\eta\omega^4}{(1-\omega^2)(1+\omega^2-\mu)+\eta\omega^4} \tag{43}$$

In the limiting case of $\omega = 1$, the outer flow will approach the case of flow between two parallel plates. Therefore, $(Nu)_{oo}$ and $(Nu)_{io}$ will approach the values of the Nusselt numbers for flow between two parallel plates whose surfaces are maintained by the heating conditions expressed in (43) if the hydraulic diameter is used in (40).

Case III

If the heat transfer coefficient at the interface is defined relative to the difference between the outer flow and the inner flow bulk temperatures, one writes

$$U_o = (T_{mo}-T_{mi})2\pi L\bar{h}_o$$

$$\text{and} \quad U_i = -(T_{mo}-T_{mi})2\pi\omega L\bar{h}_i. \tag{44}$$

Here again the heat flow rate is assumed to be positive when heat is flowing into the outer flow region.

Selecting the fluid in the outer passage as the reference fluid, the Nusselt numbers at the interface and interface surface based on their diameter's size are

$$(Nu)_i = \frac{2\omega L}{k_o}\bar{h}_i, \quad (Nu)_o = \frac{2L}{k_o}\bar{h}_o. \tag{45}$$

Equating the two forms of the rates of flows in (22) and (44), and neglecting heat generation, after some calculations one finds

$$\eta\omega^4 = -2\left(\frac{J_o/G_o}{I_{oo}}-\frac{11}{96}\omega^4\frac{\eta}{k_k}+\beta\right)(Nu)_i \tag{46}$$

$$(1-\omega^2)(1+\omega^2-\mu)+\eta\omega^4 =$$
$$-2\left(\frac{J_o/G_o}{I_{oo}}-\frac{11}{96}\omega^4\frac{\eta}{k_k}+\beta\right)(Nu)_o \tag{47}$$

where I_{oo}, β, J_o/G_o are given in (7), (18) and (29), respectively. Here again it is to be noted that $(Nu)_i$ and $(Nu)_o$ depend on the same two dimensionless parameters:

(i) The thermal conductivity of the two fluids, k_k
(ii) The heat exchange number, η.

In this case it must be noted that these Nusselt numbers do not approach any known values for the limiting cases of $\omega = 0$ and $\omega = 1$.

4. DISCUSSION OF NUMERICAL RESULTS AND CONCLUSIONS

Convective heat transfer for laminar flows in a multi-passage circular pipe subjected to an external uniform heat flux is analyzed and the results are presented in graphical form for three cases. For each case, the variation of the Nusselt numbers are plotted in two different ways as follows:

1. Selecting a numerical value for the ratio of the thermal conductivity of two fluids as $k_k = 1$, and using four fixed values for dimensionless heat exchanger number η (1, 10, -1, -10), both Nusselt numbers are plotted against the dimensionless radius of the interface for all three cases. These curves are shown in Figs. 2–7.

2. Again selecting the same numerical value of thermal conductivity ratio as $k_k = 1$, but using only one value for dimensionless inner pipe radius ω as 0.4, both Nusselt numbers are plotted against the dimensionless heat exchanger number, η for all of the three cases. These curves are shown in Figs. 8 and 9.

It must be noted that, except for the Case III, in the limiting cases of $\omega = 0$ and $\omega = 1$, the Nusselt numbers coincide with the expected value of 48/11.

The graphical results are only for one value of thermal conductivity ratio $k_k = 1$ which is characteristic of the same fluid in the inner as well as the outer passages. It is indeed easy to evaluate equations (36), (37), (41), (42), (46) and (47) numerically for any other thermal conductivity ratio.

The results presented in Figs. 2–7 have been verified for the limiting cases of $\omega = 0$ and $\omega = 1.0$. No attempt has been made to compare these results for other values

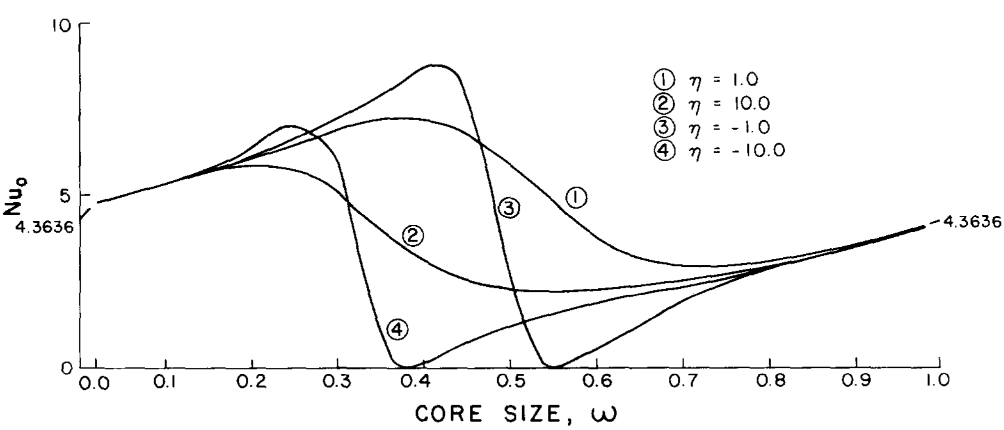

FIG. 2. Nu_o vs ω for Case I.

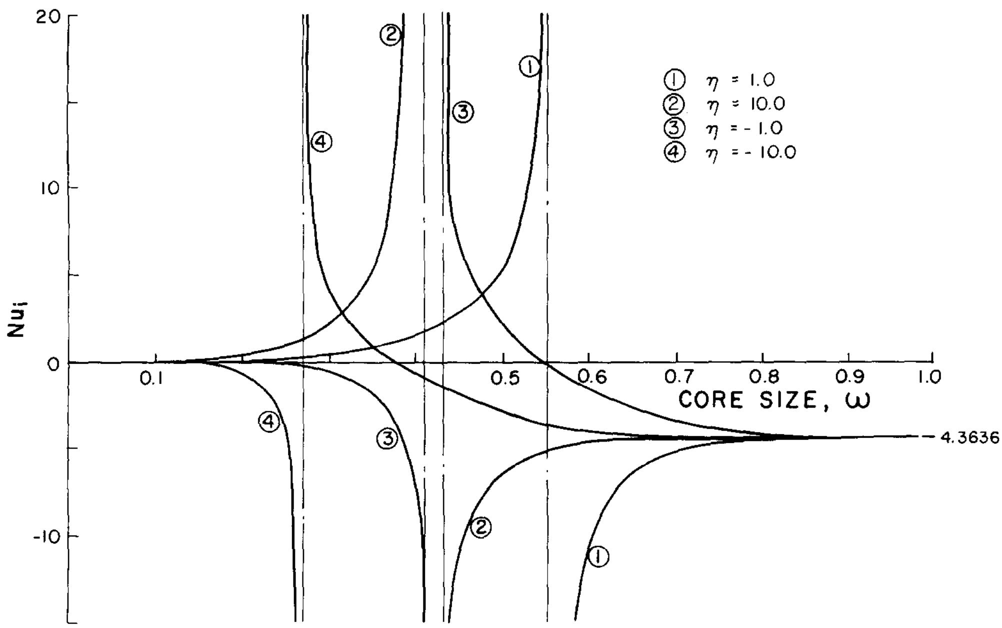

FIG. 3. Nu_i vs ω for Case I.

of ω. The numerical calculations to plot these figures are trivial and no discussion is presented. However, a discussion is presented for each of the figures.

Figure 2. The representative curves show the variation of outer wall Nusselt number defined relative to the mixed bulk temperatures of the inner and outer flows with the dimensionless radius of inner separation, ω. In both limiting cases of $\omega = 0$ and $\omega = 1$, the system simplified to a simple pipe flow with a uniform external heat flux and the Nusselt number reduces to the expected value of 48/11. In the case of parallel flow arrangement, the curves (1) and (2) indicate highest and lowest Nusselt numbers for special values of the separating wall size ω. In the case of counter-flow arrangement, the curves (3) and (4) indicate again highest and lowest Nusselt numbers for particular values of ω. However, this time for each heat exchange number η there is a special ω for which the Nusselt

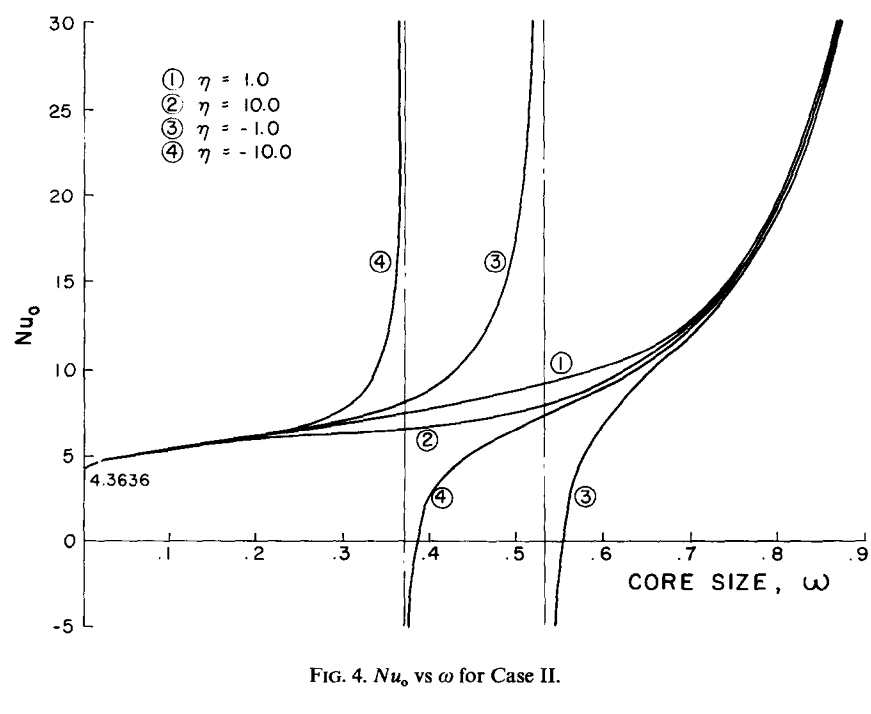

FIG. 4. Nu_o vs ω for Case II.

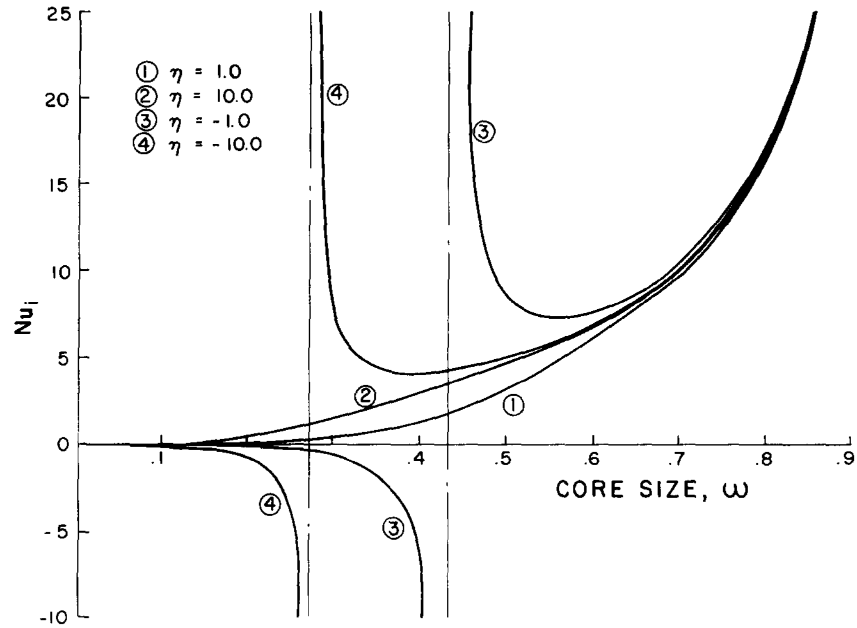

FIG. 5. Nu_i vs ω for Case II.

number vanishes. It can be noted that the Nusselt numbers at the separating wall also vanish—the curves (3) and (4) of Fig. 3—exactly for the same particular values of ω. This behavior corresponds to special counter-flow arrangements (with special η and special ω), where the thermally fully-developed conditions cannot be reached with a finite external heat flux. In that case, a uniform temperature prevails everywhere with no external heat flux.

Figure 3. For both parallel and counter-flow arrangements in the limiting case of $\omega = 1$, the outer flow disappears and the system reduces to a simple pipe where the Nusselt number at the inner surface represents the outer wall Nusselt number (48/11) of the simple pipe flow. This value is observed as the horizontal asymptote of all curves.

It is seen that all curves have a unique vertical asymptote. For example, in the case of parallel flow arrangement with $\eta = 1$, when the dimensionless radius of separation wall is smaller than 0.5498, the

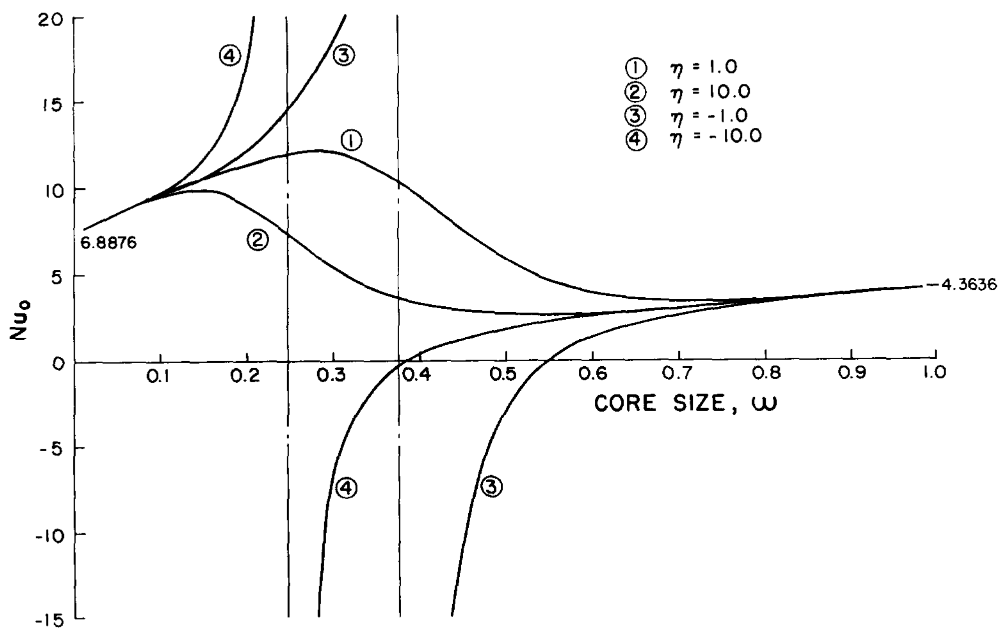

FIG. 6. Nu_o vs ω for Case III.

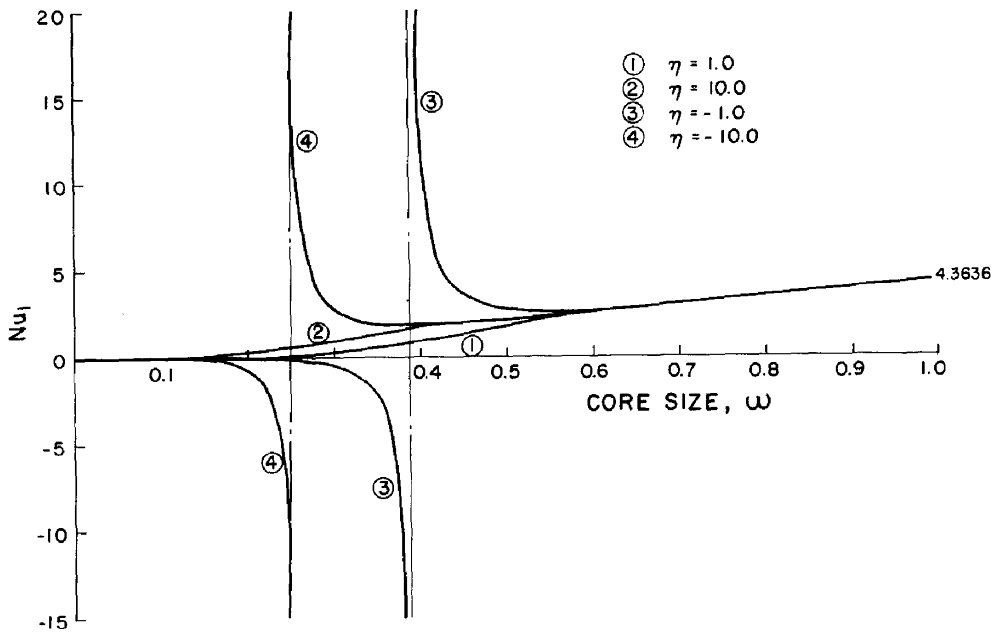

FIG. 7. Nu_i vs ω for Case III.

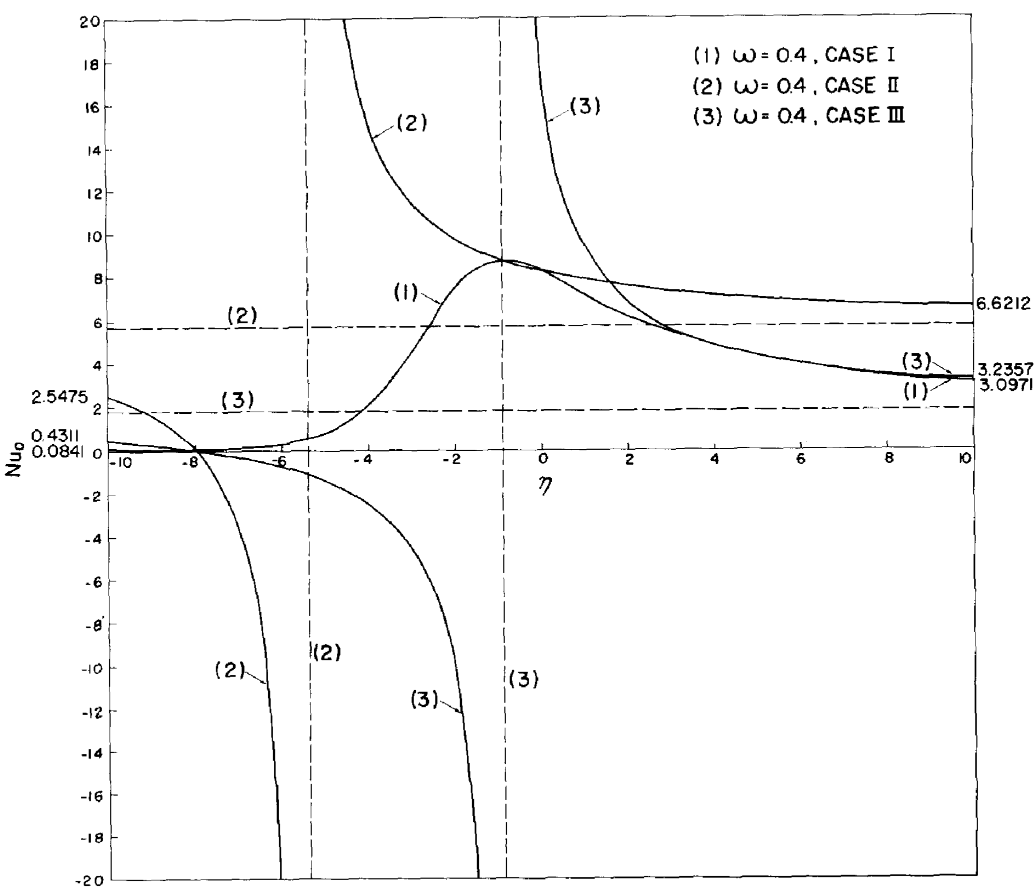

FIG. 8. Nu_o vs η for Cases I, II and III for $\omega = 0.4$.

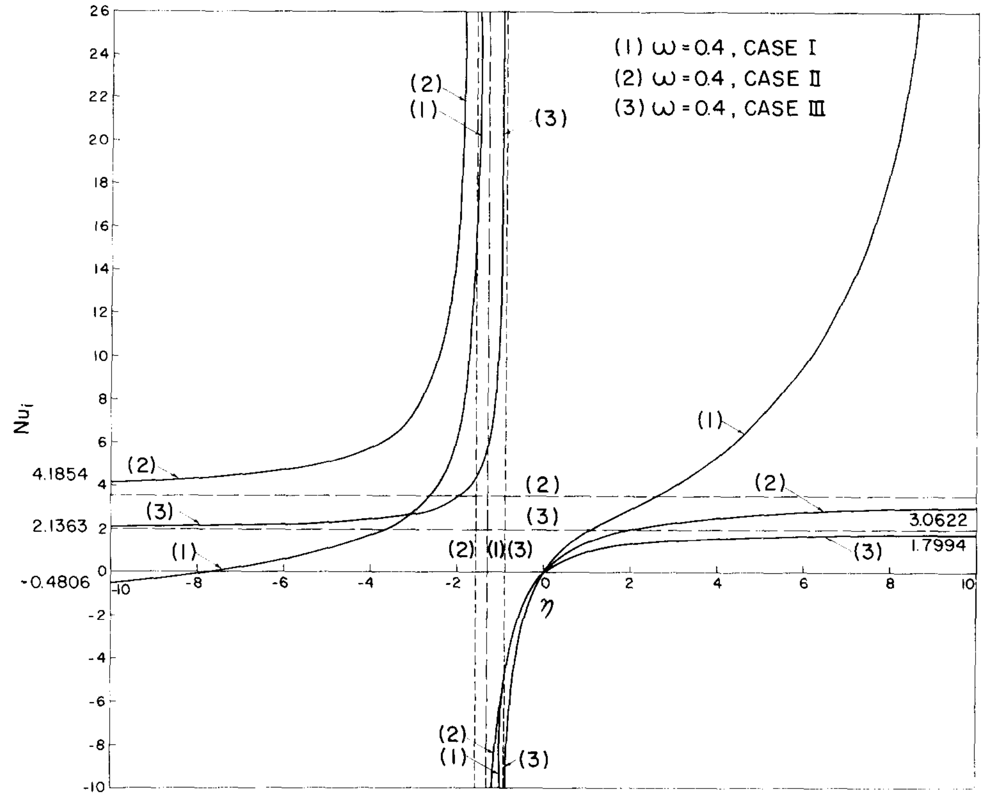

FIG. 9. Nu_i vs η for Cases I, II and III for $\omega = 0.4$.

Nusselt number at the interface and based on the total mixed bulk temperature of both flows has positive values. This means that heat flows from the inner to the outer region when the inner wall temperature is greater than the overall bulk temperature, and heat flows from the outer to the inner region when the inner wall temperature is smaller than the overall bulk temperature. When ω is larger than 0.5585, the opposite happens. That is, the heat flow direction can be related to the difference between the inner wall temperature and overall bulk temperature. In the case of counter flow arrangement, a similar property prevails. However, the opposite heat flow directions at the inner surface take place than that described for parallel flow. It is also seen that the curves (3) and (4) intersect the horizontal axis at special points. The significance of these points has been described earlier.

Figure 4. The representative curves show the variation of outer surface Nusselt number defined relative to the bulk temperature of outer flow and the variation of inner surface Nusselt number based on the bulk temperature of the inner flow with the dimensionless radius of inner separation, ω. It is seen that in the limiting case of $\omega = 0$, the system simplifies to a simple pipe flow and all the Nusselt numbers reduce to 48/11. When ω approaches 1.0, the Nusselt numbers become very large corresponding to the cases for which the bulk temperatures and the wall temperatures become equal.

In the case of counter-flow arrangements, the curves (3) and (4) have vertical asymptotes. Around these asymptotes, the Nusselt number signs change and the heat flow directions at the outer surface can be related to the differences of the bulk and wall temperatures. In addition, the counter-flow Nusselt numbers vanish at some special values of ω. The significance of these conditions was discussed in the discussion of Fig. 2.

Figure 5. On the inner separating wall two Nusselt numbers are considered. The Nusselt number on the outer face of the interface, Nu_{io}, and the Nusselt number on the inner face of the interface, Nu_{ii}. As it was indicated in Section 2, Nu_{ii} has the trivial value of 48/11. In the case of parallel flow, the curves (1) and (2) show the variation of Nu_{io} with the dimensionless radius of the interface indicating a gradual change from zero to infinity. It must be noted that due to the simplicity of geometry, the interface Nusselt numbers are defined, equation (40), on the basis of the diameter of the inner wall. Therefore, the infinitely large values of Nu_{io} for ω approaching unity are to be expected. In addition, the values of Nu_{io} for any ω are not directly comparable to the values obtained in [8] since the Nusselt numbers were based on the hydraulic diameter.

Figure 6. The representative curves show the variation of the outer wall Nusselt number defined relative to the difference between the outer and inner flow bulk temperatures with the dimensionless radius of interface ω. It must be noted that in the limiting case of $\omega = 0$, the values of the Nusselt numbers do not approach any established value. This is due to the fact that the Nusselt numbers are not defined relative to any of the bulk temperatures. In the case of parallel flow arrangement, the curves (1) and (2) indicate highest and lowest Nusselt numbers for special values of the separating wall size ω. In the case of counter-flow arrangement, the curves (3) and (4) have vertical asymptotes indicating that the Nusselt numbers, as defined in this paper, are changing their sign around these special ω values.

Figure 7. The representative curves show the variation of inner surface Nusselt number defined relative to the difference between the outer and inner flow bulk temperatures with the dimensionless radius of inner surface. Again in the limiting case of $\omega = 1$, the Nusselt numbers approach the established value of 48/11. In the case of parallel flow, the Nusselt numbers are increasing gradually from 0 to 4.3636. In the case of counter flow, the curves (3) and (4) have vertical asymptotes. The significance of vertical asymptotes was also discussed for the previous cases.

Figures 8 *and* 9. The gradual effect of heat exchange number, η, on the outer Nusselt number, Nu_o, and on the inner Nusselt number, Nu_i, for each three cases are presented for a fixed value of ω. The range of η is selected from -10 to $+10$. The Nusselt numbers at the end points, $\eta = -10$ and $\eta = 10$, thus correspond to the Nusselt numbers for these particular η values. It is seen that for Cases II and III, there is a horizontal and a vertical asymptote. Around the η values of these asymptotes, Nusselt numbers change sign indicating a heat flow direction property as discussed in the previous cases.

Acknowledgment—This material is based upon work supported by the National Science Foundation under the grant No. (R11-8305297). The authors also appreciate the contribution made by Mr Lenhat Le for the numerical computations.

REFERENCES

1. R. K. Shah and A. L. London, *Laminar Flow Forced Convection in Ducts.* Academic Press, New York (1978).
2. R. I. Soloukhin and O. G. Martynenko, Heat and mass transfer bibliography—Soviet literature, *Int. J. Heat Mass Transfer* **26**, 1771–1781 (1983).
3. E. R. G. Eckert, R. J. Goldstein, E. Pfender, W. E. Ibele, S. V. Patankar, J. W. Ramsey, T. W. Simon and N. A. Deckor, Heat transfer—A review of the 1982 literature, *Int. J. Heat Mass Transfer* **26**, 1733–1770 (1983).
4. R. E. Lundberg, W. C. Reynolds and W. M. Kays, Heat transfer with laminar flow in concentric annuli with constant and variable wall temperature with heat flux, NASA Technical Note, TN D-1972, Washington, DC (August 1963).
5. W. M. Kays and M. E. Crawford, *Convective Heat and Mass Transfer*, 2nd edn. McGraw-Hill, New York (1980).
6. E. R. G. Eckert and R. M. Drake Jr, *Analysis of Heat and Mass Transfer.* McGraw-Hill, New York (1972).
7. B. Gebhart, *Heat Transfer.* McGraw-Hill, New York (1971).
8. H. C. Topakoglu and O. A. Arnas, Convective heat transfer for steady laminar flow between two confocal elliptic pipes with longitudinal uniform wall temperature gradient, *Int. J. Heat Mass Transfer* **17**, 1487–1498 (1974).

CONVECTION THERMIQUE POUR LES ECOULEMENTS LAMINAIRES DANS UN TUYAU CIRCULAIRE MULTI-PASSAGE, SOUMIS A UN FLUX DE CHALEUR EXTERNE UNIFORME

Résumé—On analyse la convection thermique d'un écoulement hydrodynamiquement et thermiquement établi dans un tube circulaire multi-passage, soumis à un flux de chaleur externe uniforme. Les paramètres sans dimension qui caractérisent le problème sont déterminés. On obtient les expressions des nombres de Nusselt sur les parois interne et externe. Les cas limites sont reliés aux valeurs existantes. On donne les courbes représentant la variation des nombres de Nusselt avec les paramètres pertinents.

KONVEKTIVER WÄRMEÜBERGANG BEI LAMINARER STRÖMUNG IN EINEM MEHRGÄNGIGEN KREISRUNDEN ROHR BEI AUFGEPRÄGTER ÄUSSERER WÄRMESTROMDICHTE

Zusammenfassung—Der konvektive Wärmeübergang bei einer hydrodynamisch und thermisch voll entwickelten Strömung in einem mehrgängigen kreisrunden Rohr wurde bei aufgeprägter äußerer Wärmestromdichte untersucht. Die signifikanten dimensionslosen Parameter wurden bestimmt. Es werden Ausdrücke für die Nusseltzahl des inneren und äußeren Wärmeüberganges ermittelt. Die Grenzfälle werden mit vorhandenen Werten korreliert. Die dargestellten Kurvenverläufe zeigen die Abhängigkeit der Nusseltzahl von den entsprechenden Parametern.

КОНВЕКТИВНЫЙ ТЕПЛООБМЕН ПРИ ЛАМИНАРНОМ ТЕЧЕНИИ В МНОГОКАНАЛЬНОЙ КОЛЬЦЕВОЙ ТРУБЕ С РАВНОМЕРНЫМ НАГРЕВОМ ВНЕШНИМ ТЕПЛОВЫМ ПОТОКОМ

Аннотация—Анализируются характеристики конвективного теплообмена гидродинамически и термически полностью развитого течения в многоканальной кольцевой трубе при равномерном нагреве внешним тепловым потоком. Определены важные безразмерные параметры. Получены выражения для чисел Нуссельта на внутренней и внешней стенках. Рассмотрено соотношение между данными в предельных случаях и имеющимися величинами. Построены графические зависимости изменения чисел Нуссельта от соответствующих параметров.